Incumbent-Challenger and Open-Seat Elections in a Spatial Model of Political Competition

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Abstract

I compare the outcomes of incumbent-challenger and open-seat elections in a model of spatial electoral competition. Candidates are policy motivated and may differ in their non-policy attributes which are collectively referred to as valence characteristics. Candidates adopt more divergent policies in the incumbent-challenger election compared to an open-seat contest and the divergence is higher when the incumbent is at a valence disadvantage. Policy motivated incumbents benefit from an incumbency advantage in being able to choose their policy before the challenger.

The literature on spatial electoral competition is dominated by models in which office motivated candidates announce policies simultaneously.[1] I focus on two features of elections which are often overlooked − candidates may be policy motivated and announce policies at different times. Simultaneous policy announcements may be a reasonable assumption for open-seat elections, however incumbents are likely to announce policies before challengers. As such I model incumbent-challenger elections as sequential move games with the incumbent as first mover.

Politicians in my model are motivated by policy and their utility is based on the proximity of the enacted policy to their preferred policy point.[2] Winning office is not the goal in itself but rather a means to implementing policy. A candidate's desire to win comes from her desire to implement a favorable policy coupled with her desire to stop the opponent implementing a distasteful policy.

In reality politicians may be motivated by both office and policy and an advantage of Groseclose (2001) is to allow candidates to have mixed motivations. However this becomes very complicated quite quickly and for some parameterizations Groseclose resorts to simulation analysis. I adapt Grose-

[1]See e.g., Ansolabehere and Snyder (2000), Aragones and Palfrey (2002), Ashworth and Bueno de Mesquita (2009), Zakharov (2009), Krasa and Polborn (2012), Hummel (2010), Duggan (2007).

[2]As noted by Wittman (1973 & 1977), working with policy motivated candidates avoids the strange result which arises from the assumption of office motivated candidates whereby the only participants in the model who are not interested in policy are the candidates themselves.

close (2001) to allow for sequential policy announcements of policy motivated candidates. This abstraction from office motivation allows us to focus solely on the electoral effects and incentives involved with policy motivated politicians which should help improve our understanding of electoral competition.[3]

The theoretical predictions of my model are consistent with empirical studies of the US Congress which indicate policy divergence (see e.g., Poole and Rosenthal, 1984 & 1997 and McCarty et al., 2006). I show the timing of policy announcements affects the degree of policy divergence. Incumbent-challenger (sequential-move) elections generate greater policy divergence than open-seat (simultaneous-move) elections and divergence is greatest when the incumbent is at a valence disadvantage. Ansolabehere, Snyder and Stewart (2001) analyze the ideological positioning of candidates in the US House and find that incumbents are more moderate than challengers. However Burden (2004) contradicts this result in separate work which shows that incumbents are more extreme than challengers. In my model, the incumbent adopts a more extreme position than the challenger if the incumbent has a small valence advantage or a valence disadvantage, consistent with Burden (2004). However as the incumbent's valence advantage increases she may adopt a more moderate position than the challenger, consistent with Ansolabehere, Snyder and Stewart (2001).

I show that policy motivated incumbents benefit from a first-mover ad-

[3]Callander (2008) develops a model of electoral competition where candidates may be either office or policy motivated and shows that in equilibrium a majority of candidates must be policy motivated. Therefore understanding policy motivated candidates is necessary to fully understand elections.

vantage. The incumbent chooses a policy close to her preferred policy point. This causes the ideologically opposed challenger to react by adopting a relatively moderate policy to capture more votes and reduce the incumbent's probability of winning. As a result the incumbent's expected utility is high; if the incumbent wins the election she gets to implement her preferred policy but even if she loses the election the moderate policy of the challenger is not too distasteful.

Previous studies on sequential policy announcements have made contributions in the area of office motivated candidates (see e.g., Berger et al., 2000; Ingberman, 1992; Bernhardt and Ingberman, 1985; Palfrey, 1984; Weber, 1992). The comparison of incumbent-challenger and open-seat elections has received little attention either because no simultaneous move equilibrium exists in models with office motivated candidates, as in Berger et al. (2000), or if it does exist it is the same as the sequential move equilibrium, as in Bernhardt and Ingberman (1985).[4] Londregan and Romer (1993) consider sequential policy announcements where the incumbent is forced to run on fixed policy record. However the important issues in election campaigns may change over time and incumbents must react to new policy proposals or adapt their positions due to changing circumstances. As such I allow incumbents to choose their policy rather than taking it as fixed.

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[4]The non-existence of a simultaneous move equilibrium with office motivated candidates is well known and is referred to as a kind of "folk theorem" by Groseclose (2001). The advantaged candidate always wants to mimic the disadvantaged candidate whereas the disadvantaged candidate always wants to move away from the advantaged candidate.

In Anderson and Glomm (1992) candidates care about winning but also dislike compromising their own ideals.[5] However this does not capture policy motivation as the candidate gets zero utility if she loses regardless of the position of the challenger. In Anderson and Glomm (1992) there is a disadvantage to moving first if the candidates possess equal valence or the incumbent has a small valence advantage. In my model the policy motivated incumbent benefits from a first mover advantage irrespective of which candidate possesses the valence advantage.

The Model

A left-wing candidate L and a right-wing candidate R compete by choosing policies from a one dimensional policy space with candidate $c \in \{L, R\}$ choosing a policy $P_c \in [-1, 1]$. In the incumbent-challenger election, the incumbent L chooses her policy position P_L^I before the challenger. The challenger R observes the incumbent's policy and reacts by choosing a policy position P_R^C. The timing of the game is different in an open-seat election as both the left and right wing candidates simultaneously choose their policies P_L* and P_R* respectively.

Voters derive utility from a candidate's exogenous valence endowment as well as the proximity of the candidate's chosen policy platform to the voter's

[5]In a related strand of literature, Wiseman (2006) studies a model of partisan campaign support and entry deterrence where a party's utility depends on both policy and the amount of costly campaign support provided to their candidate.

ideal point. If both candidates choose the same policy platform the voter prefers the candidate with the higher valence.[6] The utility of voter i with ideal point m_i if candidate $c \in \{L, R\}$ wins is

$$U_i(P_c, v_c) = v_c - (P_c - m_i)^2 \tag{1}$$

where v_c is the valence and P_c the policy choice of candidate c.[7] The candidate who captures the median voter wins the election. The election outcome is probabilistic due to incomplete information about the median voter. The candidates do not know the median voter's exact location but they know the median voter is drawn from a continuous uniform distribution with support $[-1, 1]$. The cumulative distribution function is denoted by $F(\cdot)$.

For every combination of candidate policy choices and valence endowments, there exists a unique point on the policy space at which the voter is indifferent between the two candidates. I denote the indifferent voter as m_β. The probability that candidate L wins is the probability that the median voter lies to the left of the indifferent voter, $F(m_\beta)$. The probability that R wins is $1 - F(m_\beta)$. An expression for the indifferent voter m_β is obtained by

[6]Valence may consist of non-policy characteristics such as competence, integrity, intelligence and dedication to public service.

[7]Voter preferences are quadratic in policy as in Wiseman (2006), Ansolabehere and Snyder (2000), Adams et al. (2011) and Ashworth and Bueno de Mesquita (2009).

equating the voter utility for candidate L with that of candidate R,

$$v_L - (P_L - m_\beta)^2 = v_R - (P_R - m_\beta)^2 \tag{2}$$

Defining $v = v_L - v_R$ and solving for m_β yields the following expression,

$$m_\beta = \frac{v - P_L^2 + P_R^2}{2(P_R - P_L)} \tag{3}$$

By using equation (3) in the uniform CDF I can define L's probability of victory as follows,

$$F(m_\beta) = \frac{1}{2} - \frac{v - P_L^2 + P_R^2}{4P_L - 4P_R}, m_\beta \in [-1, 1] \tag{4}$$

Lemma 1 $\frac{\partial F(m_\beta)}{\partial v} > 0$, $\frac{\partial F(m_\beta)}{\partial P_L} > 0$ *and* $\frac{\partial F(m_\beta)}{\partial P_R} > 0$.

Proof: See appendix.

An increase in L's valence advantage leads to an increase in L's probability of victory. With higher valence L becomes a more appealing candidate which increases her probability of capturing the median voter.[8] A move by L towards the center of the policy space in the direction of the expected median voter also increases her probability of victory. Similarly, a rightward move by R away from the expected median voter increases L's chances of winning.

Candidates are policy motivated and view winning as a means to imple-

[8]This is true provided $P_L \leq P_R$, which always holds as a policy motivated left wing candidate will never choose a more right wing policy than the ideologically opposed right wing candidate.

menting policy. Following an election, the utility of candidate $c \in \{L, R\}$ with ideal policy position x_c is

$$V_c = -(P_w - x_c)^2 \tag{5}$$

where P_w is the policy location of the winning candidate. As in Groseclose (2001) I assume the candidates' ideal points are symmetric about the expected median voter. Candidate *L's* ideal point is x_L=-1 and candidate *R's* ideal point is x_R=1. If we take the view that candidates are chosen by parties then x_L and x_R can be interpreted as the ideal points of the median members from two polarized parties.

Candidate $c \in \{L, R\}$ chooses a policy position to maximize the following pre-election expected utility function,

$$EU_c = F(m_\beta)[-(P_L - x_c)^2] + (1 - F(m_\beta))[-(P_R - x_c)^2] \tag{6}$$

When $c = L$ candidate L chooses P_L to maximize equation (6). Likewise when $c = R$ candidate R chooses P_R.

Reaction Functions

I begin by assuming any valence advantage that may exist accrues to candidate L with $v = v_L - v_R \geq 0$.[9] I categorize valence as either "low" or

[9] In open-seat elections there is no loss in generality by assuming L has a valence advantage. For incumbent-challenger elections I relax this assumption later to consider elections with a valence disadvantaged incumbent.

"high". With a low valence advantage L's policy does not guarantee victory, however if valence is high enough to exceed a threshold denoted $\underline{v}$ then the equilibrium outcome results in L winning with a probability of one.

Lemma 2 *There is a valence advantage threshold* $\underline{v}$ *such that if* $v \geq \underline{v}$ *then* $F(m_\beta) = 1$ *and L guarantees victory in equilibrium in the incumbent-challenger and open-seat elections. L's policy choice always lies within* $P_L \in [-1, 1-\sqrt{v}]$.

Proof: See appendix.

If $0 < v < \underline{v}$ then interior solutions, defined as $-1 < P_L < 1-\sqrt{v}$, exist for both the incumbent-challenger and open-seat contest. Corner solutions occur when L possesses a high valence advantage ($v \geq \underline{v}$). In what follows I focus on the low valence case. This is of primary interest as neither candidate is guaranteed of victory and different outcomes are observed in the open-seat and incumbent-challenger elections. Moreover in reality close elections are generally more interesting compared to one-sided elections where the outcome is a foregone conclusion. Nonetheless the high valence case with corner solutions is interesting in itself as it allows the advantaged candidate to pursue her own policy agenda rather than that of the expected median voter while still guaranteeing victory. The high valence case is presented in a separate section later in the paper.

The first order condition of the optimization problem $\frac{\partial EU_c}{\partial P_c}$ yields the reaction function for candidate $c \in \{L, R\}$ $-$ the candidate's optimal response to each possible policy choice by their opponent. L's reaction function is,

$$R_L(P_R, v) = -\frac{4}{3} + \frac{2({P_R}^2 + 2P_R + .75v + 1)^{1/2}}{3} - \frac{P_R}{3} \tag{7}$$

and R's reaction function is,

$$R_R(P_L, v) = \frac{4}{3} - \frac{2({P_L}^2 - 2P_L - .75v + 1)^{1/2}}{3} - \frac{P_L}{3} \tag{8}$$

Proposition 1 *If* $v \leq \underline{v}$ then $\frac{\partial R_L(P_R,v)}{\partial P_R} > 0$ *and* $\frac{\partial R_R(P_L,v)}{\partial P_L} > 0$. *A move along the policy line by a candidate causes their opponent to react by moving in the same direction.* $\frac{\partial R_L(P_R,v)}{\partial v} > 0$ *and* $\frac{\partial R_R(P_L,v)}{\partial v} > 0$. *An increase in L's valence advantage causes L to moderate her position and R to move to the right.*
Proof: See appendix.

The further right R moves along the policy line the more L moderates towards the center to reduce the likelihood of R winning and implementing a distasteful right-wing policy. This is illustrated by the upward sloping reaction functions in Figure 1.

With an increased valence advantage L moderates her position in order to drive home her advantage by locating closer to her opponent. Consider two candidates who adopt similar policies but one candidate has a valence advantage. The voters observe that neither candidate differs greatly on policy and therefore valence becomes the deciding factor in the voter's decision. Therefore L is willing to move away from her ideal point towards the center of the policy space as she is compensated by a large increase in the probability of victory. Furthermore, an increase in L's valence advantage causes R to move further right. R's chances of winning are small and as such does not have much to lose by locating closer to his ideal point. The effect of an increase in valence is illustrated by the shifting reaction functions in Figure 1.

A simultaneous move Nash Equilibrium is a pair of policy positions (P_L*, P_R*) such that

$$P_L* = R_L(P_R*, v), P_R* = R_R(P_L*, v) \tag{9}$$

From this it follows that P_L* and P_R* are implicitly defined by

$$P_L* = R_L(R_R(P_L*, v), v) \tag{10}$$

$$P_R* = R_R(R_L(P_R*, v), v) \tag{11}$$

The point where the reaction functions cross is the Nash equilibrium from the simultaneous move game. This is shown in Figure 1.

In the sequential move game the incumbent L takes the challenger R's optimal response into consideration when choosing policy. Substituting R's reaction function, equation (8), into L's expected utility function gives,

$$\begin{aligned} EU_L^I(P_L^I, v) &= F(m_\beta(P_L^I, v, R_R(P_L^I, v)))[-(P_L^I - (-1))^2] \\ &+ (1 - F(m_\beta(P_L^I, v, R_R(P_L^I, v))))[-(R_R(P_L^I, v) - (-1))^2] \end{aligned} \tag{12}$$

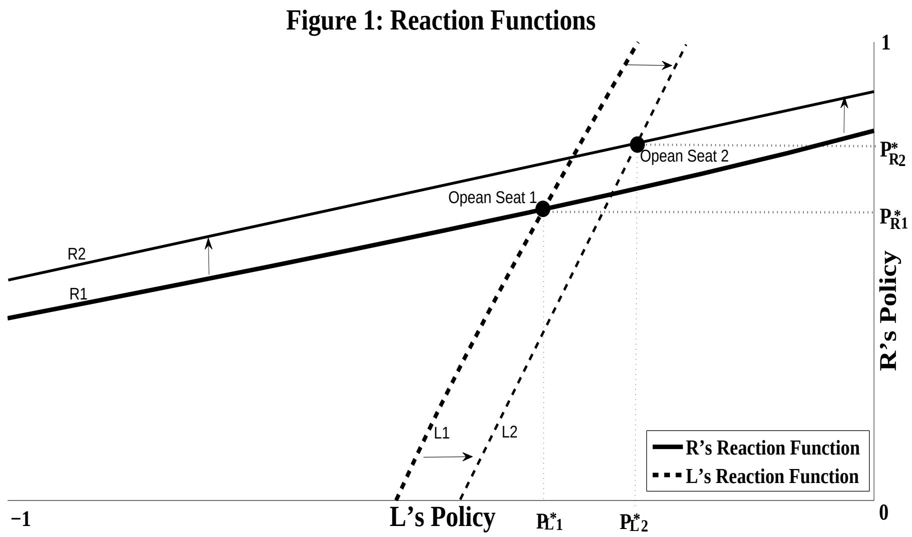

Figure 1: If $0 \leq v < \underline{v}$ a movement along the policy line by a candidate causes their opponent to react by moving in the same direction, hence the upward sloping reaction functions. An increase in L's valence advantage shifts L's reaction function from L1 to L2 and R's reaction function from R1 to R2.

and using equations (4) and (8) in equation (12) gives,

$$EU_L^I(P_L^I, v) = \frac{v}{2} - \frac{14P_L^I}{9} + \frac{vP_L^I}{6} + (\frac{2P_L^I}{9} - \frac{2}{9})\sqrt{4{P_L^I}^2 - 8P_L^I - 3v + 4}$$
$$+ \frac{(4{P_L^I}^2 - 8P_L^I - 3v + 4)^{3/2}}{54} - {P_L^I}^2 - \frac{4{P_L^I}^3}{27} - \frac{35}{27} \tag{13}$$

The utility maximization problem for L yields the first order condition,

$$\frac{\partial EU_L^I}{\partial P_L^I} = \frac{v}{6} - \frac{1}{\sqrt{4{P_L^I}^2 - 8P_L^I - 3v + 4}}(\frac{16P_L^I}{9} + \frac{8}{9} + \frac{8{P_L^I}^2}{9}) - P_L^I(2 + \frac{4}{9}P_L^I)$$
$$+ \frac{2P_L^I\sqrt{4{P_L^I}^2 - 8P_L^I - 3v + 4}}{9} - \frac{14}{9} = 0 \tag{14}$$

Implicitly defined in equation (14) is the incumbent's equilibrium policy location P_L^I for different valence endowments. When P_L^I is known then the challenger's response P_R^C is found using R's reaction function, equation (8).

Endogenizing R's response in this way enables the incumbent L to choose a point on R's reaction function. The point which L chooses is the one which yields the highest level of utility. This is the equilibrium from the incumbent-challenger election and occurs at the point where L's iso-utility curve is tangent to R's reaction function. L's iso-utility curve is concave to L's policy axis and lower iso-utility curves are associated with higher utility. The iso-utility curve and the incumbent-challenger equilibrium is shown in Figure 2.

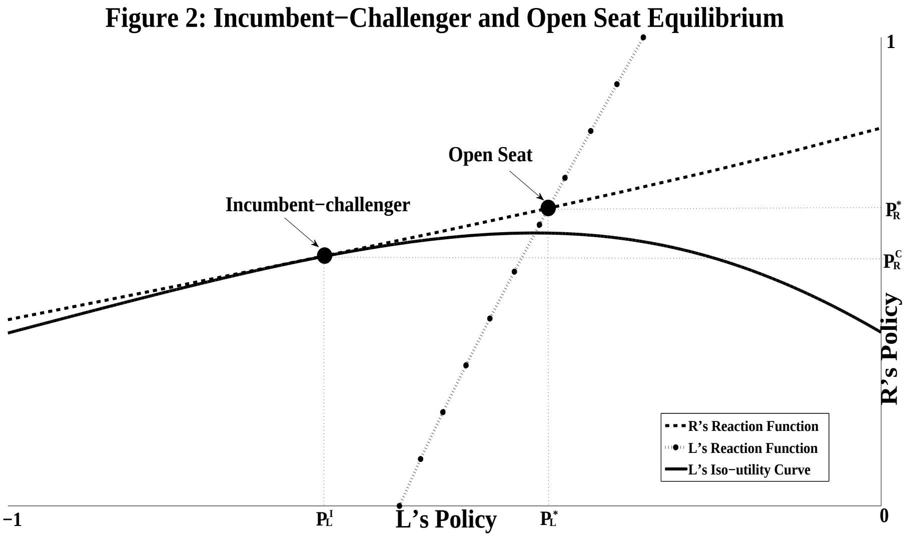

Figure 2: The equilibrium from the open-seat and the incumbent-challenger game with L as the incumbent and first-mover are shown for $0 \leq v < \underline{v}$. When L is first mover she chooses a point on R's reaction function which maximizes her utility. This is the equilibrium from the incumbent-challenger election and it occurs where L's iso-utility line is tangent to R's reaction function.

Low Valence Advantage: $0 \leq v < \underline{v}$

First consider the case where neither candidate has a valence advantage such that $v = 0$.

Proposition 2 *P_L^I and P_R^C are the equilibrium positions from the incumbent-challenger election and P_L* and P_R* are the equilibrium positions from the open-seat election. If $v = 0$ then $P_L^I = x_L = -1$, $P_R^C = \frac{1}{3}$ and $P_L* = -\frac{1}{2}$, $P_R* = \frac{1}{2}$.*

Proof: See appendix.

In the open-seat election where both candidates are equal on valence, they locate symmetrically about the center of the policy space. However in the incumbent-challenger election the incumbent as first mover strategically locates at the left of the policy space at her ideal point. As this is her preferred policy location she will obtain maximum utility if she wins. Furthermore, she knows that once her challenger R observes her left wing policy choice, he will be forced to moderate away from his ideal right wing point and locate closer to the policy center to try and win the election and stop L from implementing the left-wing policy.

Proposition 3 $\left.\frac{\partial P_L*}{\partial v}\right|_{v=0} > 0$, $\left.\frac{\partial P_R*}{\partial v}\right|_{v=0} > 0$ *and* $\left.\frac{\partial P_L^I}{\partial v}\right|_{v=0} > 0$, $\left.\frac{\partial P_R^C}{\partial v}\right|_{v=0} > 0$.

Proof: See appendix.

As L benefits from a small valence advantage she moves towards the center of the policy space. L is compensated for adopting a more moderate policy by a sizeable gain in her election probability. Initially the incumbent's policy is more extreme than the challenger's. However as the valence advantage approaches $\underline{v}$ the increasingly moderate policy of the incumbent forces the challenger to adopt an extreme position at the edge of the policy space.

The result showing the advantaged candidate in the simultaneous move game moderating towards the center has already been shown by Groseclose (2001) who refers to it as the "moderating frontrunner" result. Likewise Groseclose (2001) refers to the result in which the disadvantaged candidate in the simultaneous move game moves away from the center as the "extremist underdog" result. I show the moderating frontrunner and extremist underdog results also occur in incumbent-challenger elections.[10]

Proposition 4 *If* $0 \leq v < \underline{v}$ *then,*

(i) $P_L^I < P_L*$ and $P_R^C < P_R*$

(ii) $|P_L^I - P_R^C| > |P_L * - P_R * |$

Proof: See appendix.

The policies of both politicians lie closer to the advantaged candidate's ideological preference when that candidate is an incumbent as opposed to an

[10]In Anderson and Glomm (1992) the advantaged incumbent does not moderate but moves closer to her ideal point. Note however that in Anderson and Glomm (1992) the incumbent only cares about compromising her own ideal point $-$ if the challenger wins then the incumbent gets zero utility regardless of how extreme the challenger's position is.

open-seat contestant. While both candidates adopt positions closer to the incumbent's preferred policy, the leftward move by the incumbent is of a greater magnitude than the challenger's. The reason the challenger moderates is to increase his probability of victory and stop the incumbent winning. However there is a disutility associated with moving away from his ideal policy and this deters the challenger from moving too far left. Therefore the resulting policy divergence in the incumbent-challenger election is greater than in an open-seat election.

A Disadvantaged Incumbent

Consider elections where the incumbent is at a valence advantage such that $v = v_L - v_R < 0$.

Proposition 5 *If* $-\underline{v} < v < 0$ *then,*

(i)$P_L^I = x_L = -1$

(ii)$|P_L^I - P_R^C|$ *is higher compared to when* $v > 0$.

(iii)$P_L^I < P_L*$

Proof: See appendix.

The disadvantaged incumbent L locates at the extreme left of the policy line at her ideal point. Her reasons for doing this are twofold. Firstly, she de-emphasizes the importance of her opponents valence advantage by adopting a policy far from the opponent's. This ensures she has some positive probability of winning and implementing her preferred policy. Secondly, L's left-wing position forces R to adopt a moderate policy. The resulting policy divergence in elections with a disadvantaged incumbent is greater than elections involving a valence advantaged incumbent.

First Mover Advantage

Proposition 6 If $-\underline{v} < v < \underline{v}$ then $E(U_L^I) > E(U_L*) > E(U_L^C)$. *The incumbent benefits from moving first irrespective of whether the incumbent or challenger benefits from the valence advantage.*

Proof: See appendix.

There is a vast literature on incumbency advantage relating to direct office-holder benefits and indirect deterrence effects which boost an incumbent's probability of electoral success.[11] The first-mover advantage shown in my model points to an alternative source of incumbency advantage which relates to the incumbent's strategic ability to announce policy before the challenger.

The incumbent as first-mover can locate at, or close to, her ideal policy point and in doing so forces her opponent (the second-mover) to adopt a more moderate policy position.[12] This generates a desirable expected policy

[11]See e.g., Gelman and King (1990), Cox and Katz (1996), Levitt and Wolfram (1997), Ansolabehere and Snyder (2002), Ansolabehere et al. (2007), Lee (2008), Uppal (2010) and Trounstine (2011).

[12]I restrict attention to elections where $v < \underline{v}$. We saw from Lemma 2 that when a candidate's valence exceeds $\underline{v}$ the candidate is guaranteed to win and timing of policy choice is no longer important.

outcome for the incumbent. If she wins she can implement her own policy and if she loses the challenger's moderate policy is not too detrimental to the incumbent's utility. This prediction differs from models with office-motivated candidates where the candidate would actually prefer moving second. If a candidate has a valence advantage and is purely motivated by winning then she would like to first observe her opponent's location and guarantee victory by copying this position.

High Valence Advantage: $\mathbf{v} \geq \underline{\mathbf{v}}$

A candidate with a high valence advantage can deviate from the median voter's policy preference without fearing defeat in the election. Put simply, the candidate has the luxury of indulging her own policy agenda even if this is not representative of the majority of voters. This is referred to as ideological shirking.[13]

Note that for any given level of valence advantage there is a point at which L could locate and guarantee victory. I refer to this as a win point denoted P_L^{win}. From Lemma 2 the win point can be expressed as a function of the candidate's valence advantage, $P_L^{win} = 1 - \sqrt{v}$. This is the corner solution from the utility maximization problem. L never moves beyond the win-point as she is already guaranteed of victory.

With a low valence advantage it does not make sense to locate at the win-point as to do so would mean L adopting a very right-wing policy. However

[13]See Bender and Lott (1996) for a review of the literature on ideological shirking.

it is shown in Lemma 2 that when L has a high valence advantage such that $v \geq \underline{v}$ then she maximizes utility at her win-point in both the incumbent-challenger and open-seat elections. Her valence is high enough to allow her to locate at her win-point while maintaining a left-wing position. If the valence advantage gets high enough so as to reach the threshold $\bar{v}$ then L can locate at her ideal point at -1 and guarantee victory making this the best possible outcome of the election.

Proposition 7 *If* $\underline{v} \leq v \leq \bar{v}$ *then* $\frac{\partial P_L}{\partial v} < 0$
Proof: $\frac{\partial(1-\sqrt{v})}{\partial v} = \frac{-1}{2\sqrt{v}}$.

As L's valence advantage increases above $\underline{v}$ she can adopt increasingly left wing policy positions while still guaranteeing victory. Any further increase in valence above $\bar{v}$ has no effect.[14]

Conclusion

Simultaneous policy announcements may be a reasonable assumption for open-seat elections. However incumbent candidates are likely to announce policies before challengers making a sequential-move game more appropriate for incumbent-challenger elections. The timing of policy announcements affects the electoral outcomes. Incumbent-challenger elections generate greater policy divergence than open-seat elections and divergence is greatest when the incumbent is at a valence disadvantage.

[14]Given $P_L^{win} = 1 - \sqrt{v}$ then $\bar{v} = 4$.

When candidates are equal on valence or if the incumbent has a valence disadvantage or a slight valence advantage, then the incumbent is more extreme than the challenger. This is consistent with empirical work by Burden (2004). However with a large enough advantage the incumbent's policy becomes moderate and the challenger adopts a more extreme policy. This is consistent with Ansolabehere, Snyder and Stewart (2001).

Incumbent candidates benefit from a first-mover advantage. The incumbent's strategic ability to announce policy before the challenger generates a desirable expected policy outcome. Compared to being a candidate in an open-seat, the incumbent locates closer to her ideal policy position and this forces the challenger to moderate towards the expected median voter. If the incumbent wins the election she gets to implement her own policy. If she loses the election the challenger will implement a relatively moderate policy which is not too damaging to the incumbent's utility.

The first-mover advantage enjoyed by policy-motivated incumbents stands in contrast to models with office-motivated candidates. If a candidate is purely concerned with winning and is willing to choose whatever policy it takes to be successful, then this precludes a first-mover advantage. A valence advantaged incumbent would rather wait until the challenger has announced policy and copy that challenger's policy thereby guaranteeing victory.

Appendix

Lemma 1 $\frac{\partial F(m_\beta)}{\partial v} > 0$, $\frac{\partial F(m_\beta)}{\partial P_L} > 0$ *and* $\frac{\partial F(m_\beta)}{\partial P_R} > 0$.

Proof. $\frac{\partial F(m_\beta)}{\partial v} = \frac{-1}{(4P_L - 4P_R)} > 0$ provided $P_L < P_R$. While I make no constraints on the candidates' policy choices, apart from $P_c \in [-1, 1]$, it is always the case that $P_L < P_R$. The policy motivated left wing candidate never chooses a policy further right than the ideologically opposed right wing candidate. $\frac{\partial F(m_\beta)}{\partial P_L} = \frac{v}{4(P_L - P_R)^2} + \frac{1}{4} > 0$. $\frac{\partial F(m_\beta)}{\partial P_R} = \frac{1}{4} - \frac{v}{4(P_L - P_R)^2} > 0$ if $v < (P_L - P_R)^2$. Given $m_\beta = \dfrac{v - P_L^2 + P_R^2}{2(P_R - P_L)}$ it is straightforward to show that the inequality $v < (P_L - P_R)^2$ is the same as $P_R > m_\beta$. It cannot be that $P_R < m_\beta$ because then $\frac{\partial F(m_\beta)}{\partial P_R} < 0$ meaning that R could choose a more preferable right wing position while at the same time increasing her probability of victory. As such R always locates to the right of the indifferent voter and therefore $\frac{\partial F(m_\beta)}{\partial P_R} = \frac{1}{4} - \frac{v}{4(P_L - P_R)^2} > 0$.

Lemma 2 *There is a valence advantage threshold* $\underline{v}$ *such that if* $v \geq \underline{v}$ *then* $F(m_\beta) = 1$ *and* L *guarantees victory in equilibrium in the incumbent-challenger and open-seat elections.* L*'s policy choice always lies within* $P_L \in [-1, 1 - \sqrt{v}]$.

Proof. L has a 100 percent probability of victory when $m_\beta = 1$. From Lemma 1 we know that R never locates to the left of the indifferent voter so if $m_\beta = 1$ then $P_R = 1$. Using $P_R = m\beta = 1$ in equation (4) gives the policy point which L can locate and guarantee victory which is a function of valence, $P_L^{win} = 1 - \sqrt{v}$. Clearly L will never move further right beyond this point as she is already guaranteed of victory so therefore $P_L \in [-1, 1 - \sqrt{v}]$.

Using $P_R = 1$ in L's utility function and taking the first order condition $\frac{\partial EU_L}{\partial P_L}$ yields $\frac{v}{4} - \frac{5P_L}{2} - \frac{3P_L^2}{4} - \frac{3}{4} = 0$. Using $P_L = 1 - \sqrt{v}$ and solving for v gives $v = 1.37$. Therefore when $v = 1.37$, L locates at $P_L = 1 - \sqrt{1.37} = -0.17$ forcing R to locate at the edge of the policy space at $P_R = 1$. When $v \geq 1.37$ then L is guaranteed victory in equilibrium in both open-seat and incumbent-challenger elections so $\underline{v} = 1.37$.

Proposition 1 *If* $v \leq \underline{v}$ *then* $\frac{\partial R_L(P_R,v)}{\partial P_R} > 0$ *and* $\frac{\partial R_R(P_L,v)}{\partial P_L} > 0$. *A move along the policy line by a candidate causes their opponent to react by moving in the same direction.* $\frac{\partial R_L(P_R,v)}{\partial v} > 0$ *and* $\frac{\partial R_R(P_L,v)}{\partial v} > 0$. *An increase in* L*'s valence advantage causes* L *to moderate her position and* R *to move to the right.*

Proof: $\frac{\partial R_L(P_R,v)}{\partial P_R} = \frac{4P_R+4}{3\sqrt{4P_R^2+8P_R+3v+4}} - \frac{1}{3} > 0$ if $v < \underline{v}$. $\frac{\partial R_R(P_L,v)}{\partial P_L} = -\frac{4P_L-4}{3\sqrt{4P_L^2-8P_L-3v+4}} - \frac{1}{3} > 0$ if $v < \underline{v}$ and $P_L \in [-1, 0]$. I do not impose any constraints such that the left wing candidate is forced to implement a left wing policy (where $P_L \in [-1, 0]$) however this always occurs. As $P_L < P_R$ then if L ever chose a right wing policy she could always improve by moving leftwards to the centre and in doing so adopt a more preferable policy position while also improving her chances of winning. $\frac{\partial R_L(P_R,v)}{\partial v} = \frac{1}{2\sqrt{4P_R^2+8P_R+3v+4}} > 0$ and $\frac{\partial R_R(P_L,v)}{\partial v} = \frac{1}{2\sqrt{4P_L^2-8P_L-3v+4}} > 0$.

Proposition 2 P_L^I *and* P_R^C *are the equilibrium positions from the incumbent-challenger election and* P_L* *and* P_R* *are the equilibrium positions from the open-seat election. If* $v = 0$ *then* $P_L^I = x_L = -1$, $P_R^C = \frac{1}{3}$ *and* $P_L* = -\frac{1}{2}$, $P_R* = \frac{1}{2}$.

Proof: If $v = 0$ the reaction functions simplify to $R_L(P_R, v) = \frac{-2+P_R}{3}$ and

$R_R(P_L, v) = \frac{2+P_L}{3}$. Solving the equations simultaneously gives the policy locations for the open-seat election as $P_L* = -0.5$ and $P_R* = 0.5$. For the incumbent-challenger election I substitute $v = 0$ into the incumbent's first order condition, equation (14), and solve to give $P_L^I = -1$. Using this in R's reaction function gives $R_R(P_L, v) = \frac{2+(-1)}{3} = \frac{1}{3}$ so $P_R^C = \frac{1}{3}$.

Proposition 3 $\left.\frac{\partial P_L*}{\partial v}\right|_{v=0} > 0$, $\left.\frac{\partial P_R*}{\partial v}\right|_{v=0} > 0$ *and* $\left.\frac{\partial P_L^I}{\partial v}\right|_{v=0} > 0$, $\left.\frac{\partial P_R^C}{\partial v}\right|_{v=0} > 0$.

Proof: From equations (9) and (10), $P_L* = R_L(P_R*, v) = R_L(R_R(P_L*, v), v)$. Taking the partial derivative with respect to v gives $\frac{\partial P_L*}{\partial v} = \frac{\partial R_L}{\partial R_R}\frac{\partial R_R}{\partial P_L*}\frac{\partial P_L*}{\partial v} + \frac{\partial R_L}{\partial R_R}\frac{\partial R_R}{\partial v} + \frac{\partial R_L}{\partial v}$. Solving for $\frac{\partial P_L*}{\partial v}$ gives the expression, $\frac{\partial P_L*}{\partial v} = \frac{\frac{\partial R_L}{\partial P_R}\frac{\partial R_R}{\partial v}+\frac{\partial R_L}{\partial v}}{1-\frac{\partial R_L}{\partial P_R}\frac{\partial R_R}{\partial P_L}}$. I determine the sign of $\left.\frac{\partial P_L*}{\partial v}\right|_{v=0}$ by analyzing the RHS terms. $\left.\frac{\partial R_L}{\partial P_R}\right|_{v=0} = \frac{2P_R+2}{3\sqrt{P_R^2+2P_R+1}} - \frac{1}{3} > 0$. $\left.\frac{\partial R_R}{\partial v}\right|_{v=0} = \frac{1}{4\sqrt{P_L^2-2P_L+1}} > 0$. $\left.\frac{\partial R_L}{\partial v}\right|_{v=0} = \frac{1}{4\sqrt{P_R^2+2P_R+1}} > 0$. $\left.\frac{\partial R_R}{\partial P_L}\right|_{v=0} = -\frac{2P_L-2}{3\sqrt{P_L^2-2P_L+1}} - \frac{1}{3} > 0$. Finally, $\frac{\partial R_L}{\partial P_R}\frac{\partial R_R}{\partial P_L} < 1$ so $1 - \frac{\partial R_L}{\partial P_R}\frac{\partial R_R}{\partial P_L} > 0$ and therefore $\left.\frac{\partial P_L*}{\partial v}\right|_{v=0} > 0$. In the same way it can be shown that $\left.\frac{\partial P_R*}{\partial v}\right|_{v=0} > 0$. For the incumbent-challenger election recall from Proposition 2 that when $v = 0$, $P_L^I = -1$. Using this in the incumbent's first order condition, equation (14), gives $\left.\frac{\partial EU_L^I}{\partial P_L^I}\right|_{P_L^I=-1} = \frac{1}{6}(v(\frac{4}{\sqrt{16-3v}}+1))$. Clearly when $v = 0$ this equation equals zero as utility is maximized. However notice that when v goes from zero to a positive value we have $\frac{\partial EU_L^I}{\partial P_L^I} > 0$ meaning candidate L moves to the right to increase her utility. Therefore $\left.\frac{\partial P_L^I}{\partial v}\right|_{v=0} > 0$. We have already seen that $\frac{\partial R_R}{\partial P_L} > 0$ so as the incumbent L moves right so too does the challenger R and as such $\left.\frac{\partial P_R^C}{\partial v}\right|_{v=0} > 0$.

Proposition 4 *If* $0 \leq v < \underline{v}$ *then,*

(i) $P_L^I < P_L*$ and $P_R^C < P_R*$

(ii) $|P_L^I - P_R^C| > |P_L * - P_R * |$

Proof: (i) The incumbent's expected utility function is written as $EU_L^I(P_L^I, R_R^C(P_L^I, v), v)$. The first order condition for the expected utility optimization problem is $\frac{\partial EU_L^I}{\partial P_L^I} = \frac{\partial EU_L^I}{\partial P_L^I} + \frac{\partial EU_L^I}{\partial R_R^C}\frac{\partial R_R^C}{\partial P_L^I}$. Evaluating the FOC at the open-seat equilibrium position P_L* means that the first term on the RHS of the FOC satisfies L's open-seat utility optimization problem, $\left.\frac{\partial EU_L^I}{\partial P_L^I}\right|_{P_L*}$. Therefore the sign of the second term, $\frac{\partial EU_L^I}{\partial R_R^C}\frac{\partial R_R^C}{\partial P_L^I}$ tells us whether, as an incumbent, L can improve her utility by locating at a point which is different to the open-seat election. $\frac{\partial EU_L^I}{\partial R_R^C} = -\frac{{P_L^I}^2}{4} + \frac{P_L^I P_R^C}{2} + \frac{3{P_R^C}^2}{4} + \frac{v}{4} - 1 < 0$ if $-1 \leq P_L^I < 1 - \sqrt{v}$. In the proof of Proposition 1 I show that $\frac{\partial R_R}{\partial P_L} > 0$. Therefore $\frac{\partial EU_L^I}{\partial R_R^C}\frac{\partial R_R^C}{\partial P_L^I} < 0$ meaning when L is the incumbent she does better by locating to the left of her open-seat position so $P_L^I < P_L*$. From Proposition 1, $\frac{\partial R_R(P_L,v)}{\partial P_L} > 0$ meaning that $P_L^I < P_L*$ implies $P_R^C < P_R*$. (ii) It is straightforward to show that $0 < \frac{\partial R_R}{\partial P_L} < 1$ so a move to the left by L causes R to move to the left by a smaller amount. As such, given $P_L^I < P_L*$ it must be that $|P_L^I - P_R^C| > |P_L * - P_R * |$

Proposition 5 *If* $-\underline{v} < v < 0$ *then,*

(i)$P_L^I = x_L = -1$

(ii)$|P_L^I - P_R^C|$ *is higher compared to when* $v = v_L - v_R > 0$.

(iii)$P_L^I < P_L*$

Proof. If $v = 0$ then $P_L^I = -1$. The incumbent's expected utility function is $EU_L^I(P_L^I, R_R^C(P_L^I, v), v)$. From this, $\left.\frac{\partial EU_L^I}{\partial P_L^I}\right|_{P_L^I=-1} = \frac{v}{6} + \frac{32}{9\sqrt{16-3v}} - \frac{2\sqrt{16-3v}}{9}$. In this equation $v = v_L - v_R$ so when the incumbent is at a disadvantage $v < 0$. If $v < 0$ then $\left.\frac{\partial EU_L^I}{\partial P_L^I}\right|_{P_L^I=-1} < 0$. As the incumbent becomes disadvantaged she would like to move further left than $P_L^I = -1$ however $P_L \in [-1, 1 - \sqrt{v}]$. Therefore L stays at the extreme left of the policy line. For (ii), note that $\frac{\partial R_R}{\partial P_L^I} = \frac{4P_R^C+4}{3\sqrt{4{P_R^C}^2+8P_R^C+3v+4}} - \frac{1}{3}$ giving $0 < \frac{\partial R_R^C}{\partial P_L^I} < 1$. A move to the left by L causes R to move left by less than L. Given that the disadvantaged incumbent locates further left compared to when she is advantaged, it is the case that $|P_L^{inc} - P_R^{cha}|$ is higher for $v = v_L - v_R < 0$ compared to when $v = v_L - v_R > 0$. For (iii) note that Proposition 3 shows $\frac{\partial P_L*}{\partial v} = \frac{\frac{\partial R_L}{\partial P_R}\frac{\partial R_R}{\partial v} + \frac{\partial R_L}{\partial v}}{1 - \frac{\partial R_L}{\partial P_R}\frac{\partial R_R}{\partial P_L}}$. I determine the sign of $\frac{\partial P_L*}{\partial v}$ by analyzing the RHS terms of this equation. $\frac{\partial R_L}{\partial P_R} = \frac{2P_R+2}{3\sqrt{P_R^2+2P_R+.75v+1}} - \frac{1}{3} > 0$, $\frac{\partial R_R}{\partial v} = \frac{1}{4\sqrt{P_L^2-2P_L-.75v+1}} > 0$, $\frac{\partial R_L}{\partial v} = \frac{1}{4\sqrt{P_R^2+2P_R+.75v+1}} > 0$ when $-\underline{v} < v < 0$. $\frac{\partial \widehat{P_L}}{\partial P_R}\frac{\partial \widehat{P_R}}{\partial P_L} < 1$ so $1 - \frac{\partial \widehat{P_L}}{\partial P_R}\frac{\partial \widehat{P_R}}{\partial P_L} > 0$ and therefore $\frac{\partial P_L*}{\partial v} > 0$ when $-\underline{v} < v < 0$. As such $P_L^I < P_L*$.

Proposition 6 If $-\underline{v} < v < \underline{v}$ then $E(U_L^I) > E(U_L*) > E(U_L^C)$. *The incumbent benefits from moving first irrespective of whether the incumbent or challenger benefits from the valence advantage.*

Proof: This follows directly from Propositions 2, 4 and 5. The incumbent can always do at least as well as if she were a candidate in an open seat. However the incumbent as first-mover chooses a policy position further left than the open-seat equilibrium in order to gain a higher expected utility. Given the reaction functions are upward sloping and L's iso-utility curve is concave to her policy axis and lower iso-utility curves are associated with higher utility, it follows that if $P_L^I < P_L* < P_L^C$ then $E(U_L^I) > E(U_L*) > E(U_L^C)$.

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